6.6/24

$$\star = \int_{-\sqrt{2}}^{-2/\sqrt{3}} \frac{1}{x\sqrt{x^2-1}}\,dx$$

a) what is $\int \frac{1}{x\sqrt{x^2-1}}\,dx$?

$\int \frac{1}{x\sqrt{x^2-1}}\,dx =$

$\sec^{-1}|x| + C$

→ look on pg 380

→ try u-substitution

→ ask someone
- wolframalpha.com
- each other
- me

b) $\star = \sec^{-1}|x| \Big|_{-\sqrt{2}}^{\frac{-2}{\sqrt{3}}} = \star$

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$\int_a^b f(x)\,dx = F(b) - F(a)$

where $F(x)$ is an antiderivative of $f(x)$

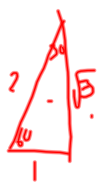
c) what is $\sec^{-1}\left|\frac{-2}{\sqrt{3}}\right|$ and $\sec^{-1}\left|-\sqrt{2}\right|$

$= \sec^{-1} \frac{2}{\sqrt{3}}$

$=$ angle whose secant is $\frac{2}{\sqrt{3}}$

$=$ angle whose cosine is $\frac{\sqrt{3}}{2}$

$= \frac{\pi}{6}$



$= \sec^{-1}\sqrt{2}$

$=$ angle whose secant is $\sqrt{2}$

$=$ angle whose cosine is $\frac{1}{\sqrt{2}}$

$= \frac{\pi}{4}$

d) $\star\ \sec^{-1}|x| \Big|_{-\sqrt{2}}^{\frac{-2}{\sqrt{3}}} = \frac{\pi}{6} - \frac{\pi}{4} = \frac{-\pi}{12}$

1a)

find area (use Def Int)

$$\int_0^2 2-x\,dx$$

$$=\left(2x-\frac{x^2}{2}\right)\Big|_0^2$$

$$=\left(2(2)-\frac{2^2}{2}\right)-\left(2\cdot 0-\frac{0^2}{2}\right)$$

$$=4-2=2$$

$$\int 2-x\,dx$$

$$=2x-\frac{x^2}{2}+C$$

$$2=2x^0$$

cheat/check

$$A_\Delta=\frac{1}{2}bh$$

$$=\frac{1}{2}(2)(2)=2$$

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6.6

3) Find area under $y=f(x)=x^3$ over $[2,3]$

"Signed"

$$\text{Area} = \int_2^3 x^3\,dx$$

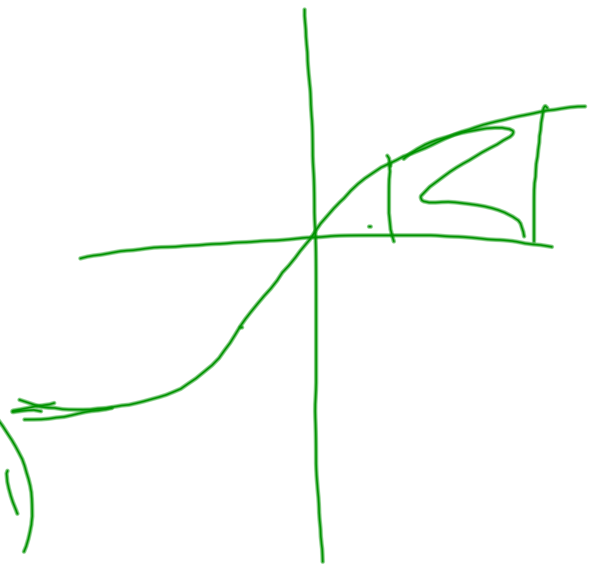
$$= \frac{x^4}{4}\Big|_2^3 = \frac{3^4}{4}-\frac{2^4}{4} = \frac{81}{4}-\frac{16}{4} = \frac{65}{4} = 16\tfrac{1}{4}$$

66/6 $f(x) = x^{-3/5}$ [1, 4]

$$\text{Area} = \int_1^4 x^{-\frac{3}{5}}\,dx$$

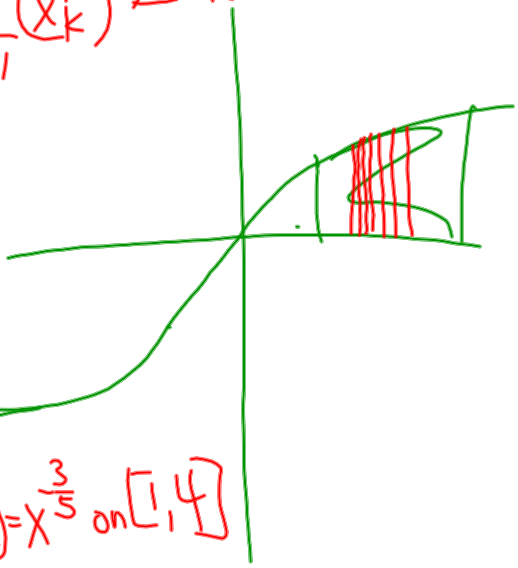
$$= \frac{5}{2}x^{\frac{2}{5}}\Big|_1^4$$

$$= \frac{5}{2}\sqrt[5]{16} - \frac{5}{2} = \frac{5}{2}\left(\sqrt[5]{16} - 1\right)$$



6b/6 $f(x)=x^{-3/5}$ $[1,4]$

$$\text{Area}=\int_1^4 x^{-\frac{3}{5}}\,dx=\lim_{\max\Delta x_k\to 0}\sum_{k=1}^{n}(x_k^*)^{\frac{3}{5}}\Delta x_k$$



Area = Sum of all heights of $f(x)=x^{\frac{3}{5}}$ over $x=1$ to $x=4$

= Accumulation of all $f(x)=x^{\frac{3}{5}}$ on $[1,4]$

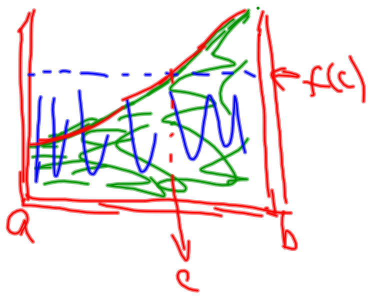
= REALLY FANCY multiplication of the constantly varying $f(x)$ Times the width of $3.\ ([4-1])$

Mean Value Theorem for Integrals

f(x) continuous.

there is ALWAYS a c in [a,b]

where $\int_a^b f(x)\,dx = f(c)\,(b-a)$



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The average value of a (continuous function) over $[a,b]$

is:

$$\text{Ave}(f(x)) = \frac{\int_a^b f(x)\,dx}{b-a}$$

Find the average value of $y = x^2$ over $[1,4]$.
find the "c" guaranteed by the mean value thm for Integrals.

$$\text{Ave value} = \frac{1}{4-1}\int_1^4 x^2\,dx = \frac{1}{3}\left(\frac{x^3}{3}\Big|_1^4\right) = \frac{1}{3}\left(\frac{64}{3} - \frac{1}{3}\right)$$

$$= \int_1^4 \frac{x^2}{3}\,dx$$

$$= \frac{1}{3}(21) = 7$$

MVTI

$f(c) = 7$

i.e. $x^2 = 7$

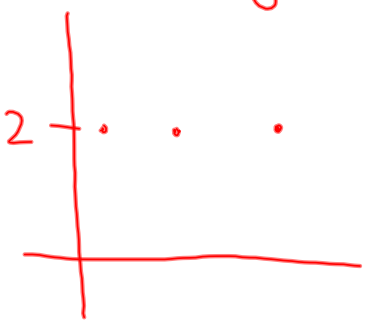
$x = \pm\sqrt{7}$ ...... the c that is guaranteed on $[1,4]$ has to be $+\sqrt{7}$

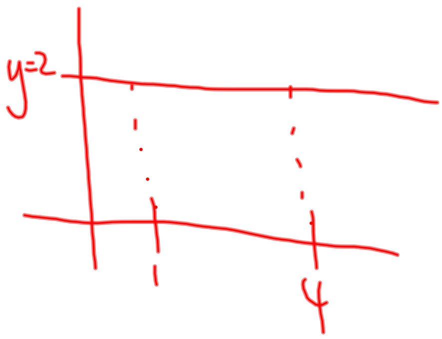
Average (in middle school)

Average value of a function [continuous]

$$\frac{\text{finite \# of things to sum}}{\text{\# of things}}$$

$$\frac{\text{infinite sum (given by definite integral)}}{\text{width of the entire interval}}$$





6.6/59-62

read rest of 6.6

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